# EE 230

## Lecture 4

**Background Materials**

Transfer Functions

Test Equipment in the Laboratory

# Quiz 3

If the input to a system is a sinusoid at 1KHz and if the output is given by the following expression, what is the THD?

$$V_{OUT} = 2\sin(2000\pi t) + 0.1\sin(4000\pi t + 45^o) + 0.05\sin(10000\pi t + 120^o)$$

# And the number is ?

1 3 8

5 4

?

2 6

9 7

# Quiz 3

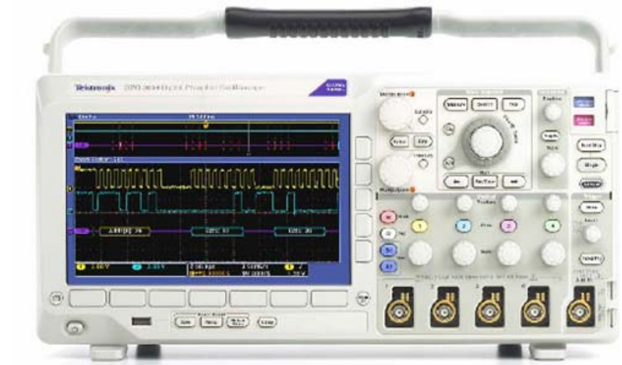
If the input to a system is a sinusoid at 1KHz and if the output is given by the following expression, what is the THD in % (based upon power)?

$$V_{OUT} = 2\sin(2000\pi t) + 0.1\sin(4000\pi t + 45^o) + 0.05\sin(10000\pi t + 120^o)$$

$$\text{THD} = \frac{\sum_{k=2}^{\infty} A_k^2}{A_1^2} \bullet 100\%$$

$$\text{THD} = \frac{0.1^2 + .05^2}{2^2} \bullet 100\% \simeq 0.31\%$$

# Test Equipment in the EE 230 Laboratory



(Plus computer, oven, software)

# Whats inside/on this equipment?

- Computer (except maybe dc power supply)
- Some analog circuitry
- Software
- Knobs/Buttons
- Computer Interface

Test equipment is becoming very powerful

Seldom need most of the capabilities of the equipment

Versatility and flexibility makes basic (and most used) operation a little more difficult to learn

# Agilent 33220A 20 MHz Function/Arbitrary Waveform Generator

# Agilent 33220A 20 MHz Function/Arbitrary Waveform Generator


User's Guide

Publication Number 33220-90002 (*order as 33220-90100 manual set*)
Edition 4, May 2007



Agilent 33220A
20 MHz Function/
Arbitrary Waveform Generator

362 Pages

# Agilent 33220A 20 MHz Function/Arbitrary Waveform Generator

## WAVEFORMS

| | |
|---|---|
| Standard | Sine, Square, Ramp, Triangle, Pulse, Noise, DC |
| Built-in arbitrary | Exponential rise, Exponential fall, Negative ramp, Sin(x)/x, Cardiac |

## WAVEFORM CHARACTERISTICS

**Sine**

| | | |
|---|---|---|
| Frequency Range | 1 µHz to 20 MHz | |
| Amplitude Flatness[1,2] | (relative to 1 kHz) | |
| | < 100 kHz | 0.1 dB |
| | 100 kHz to 5 MHz | 0.15 dB |
| | 5 MHz to 20 MHz | 0.3 dB |
| Harmonic distortion[2,3] | < 1 $V_{PP}$ | ≥ 1 $V_{PP}$ |
| DC to 20 kHz | -70 dBc | -70 dBc |
| 20 kHz to 100 kHz | -65 dBc | -60 dBc |
| 100 kHz to 1 MHz | -50 dBc | -45 dBc |
| 1 MHz to 20 MHz | -40 dBc | -35 dBc |
| Total harmonic distortion[2,3] | | |
| DC to 20 kHz | 0.04% | |
| Spurious (non-harmonic)[2,4] | | |
| DC to 1 MHz | -70 dBc | |
| 1 MHz to 20 MHz | -70 dBc + 6 dB/octave | |
| Phase noise (10 kHz offset) | -115 dBc / Hz, typical | |

## COMMON CHARACTERISTICS

| | |
|---|---|
| **Frequency** | |
| Resolution | 1 µHz |
| **Amplitude** | |
| Range | 10 $mV_{PP}$ to 10 $V_{PP}$ into 50Ω<br>20 $mV_{PP}$ to 20 $V_{PP}$ into open circuit |
| Accuracy[1,2] (at 1 kHz) | ± 1% of setting ± 1 $mV_{PP}$ |
| Units | $V_{PP}$, $V_{rms}$, dBm |
| Resolution | 4 digits |
| **DC Offset** | |
| Range (peak AC + DC) | ± 5 V into 50Ω<br>± 10 V into open circuit |
| Accuracy[1,2] | ± 2% of offset setting<br>± 0.5% of amplitude ± 2 mV |
| Resolution | 4 digits |
| **Main Output** | |
| Impedance | 50 Ω typical |
| Isolation | 42 Vpk maximum to earth |
| Protection | Short-circuit protected, overload automatically disables main output |
| **Internal Frequency Reference** | |
| Accuracy[5] | ± 10 ppm in 90 days<br>± 20 ppm in 1 year |
| **External Frequency Reference (Option 001)** | |

# Output Termination

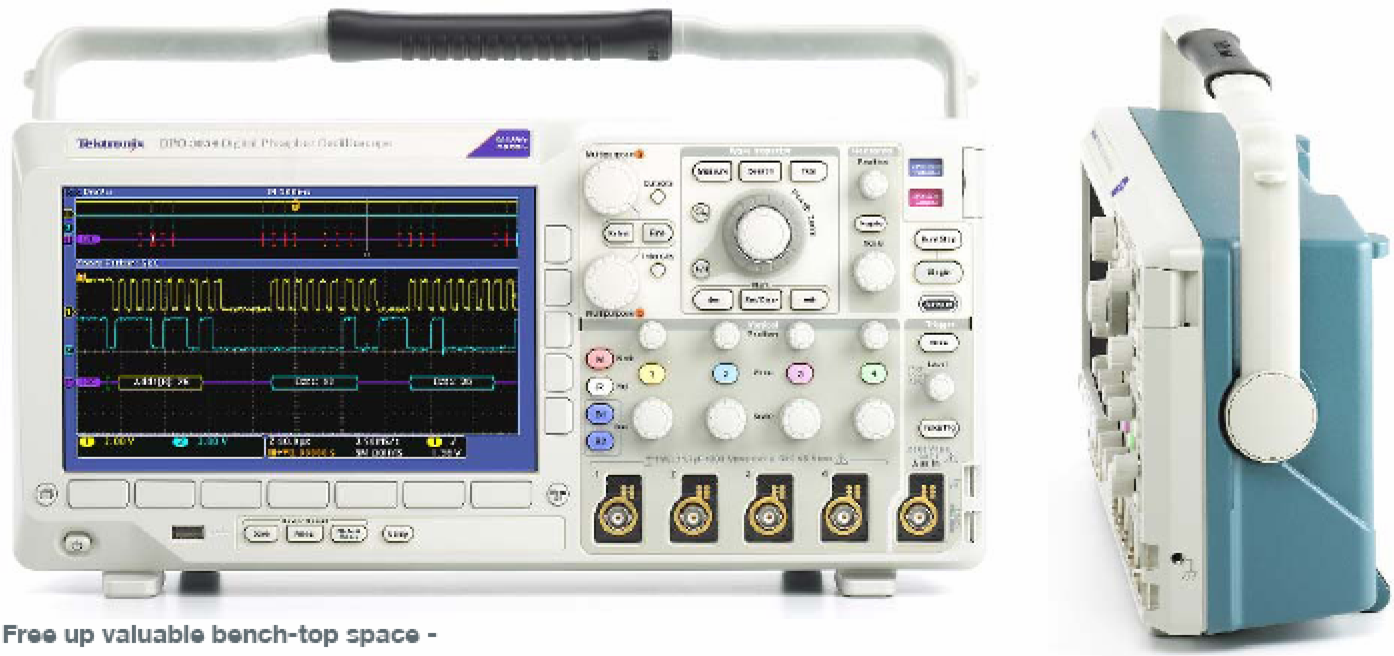

*Applies to output amplitude and offset voltage only*. The Agilent 33220A has a fixed series output impedance of 50 ohms to the front-panel *Output* connector. If the actual load impedance is different than the value specified, the displayed amplitude and offset levels will be incorrect.

- Output termination: 1Ω to 10 kΩ, or Infinite. The default is 50Ω. The message line at the top of the display calls attention to output termination settings other than 50Ω.
- The output termination setting is stored in *non-volatile* memory and *does not* change when power has been off or after a remote interface reset (assuming the Power On state is set to "default").
- If you specify a 50-ohm termination but are actually terminating into an open circuit, the actual output will be *twice* the value specified. For example, if you set the offset to 100 mVdc (and specify a 50-ohm load) but are terminating the output into an open circuit, the actual offset will be 200 mVdc.

- If you change the output termination setting, the displayed output amplitude and offset levels are automatically adjusted (no error will be generated). For example, if you set the amplitude to 10 Vpp and then change the output termination from 50 ohms to “high impedance”, the amplitude displayed on the function generator’s front-panel will *double* to 20 Vpp. If you change from “high impedance” to 50 ohms, the displayed amplitude will drop in half.

# Tektronix DPO3000 Series Oscilloscopes

## Feature-rich tools for debugging mixed signal designs



**Free up valuable bench-top space -**

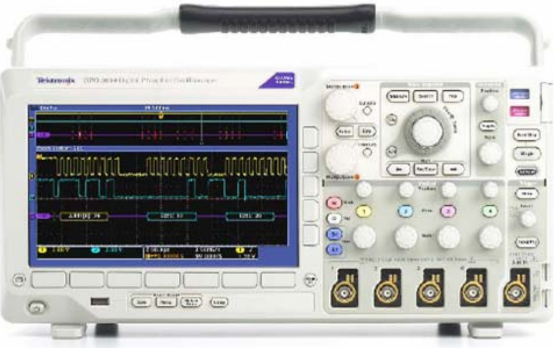
DPO3000 Series
Digital Phosphor Oscilloscopes

User Manual

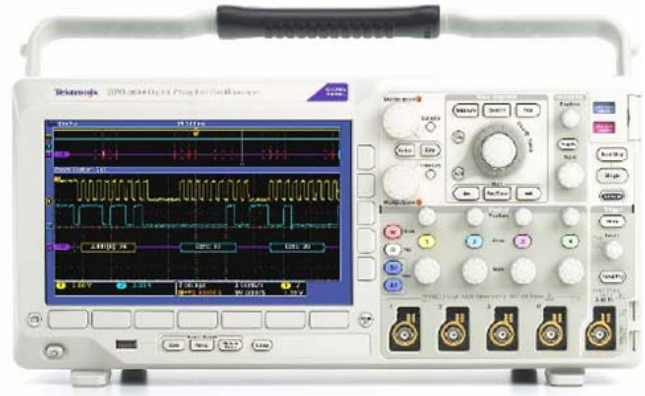
Tektronix

www.tektronix.com

071-2410-01



302 Pages



## ▸ Characteristics

| Vertical System | DPO3012 | DPO3014 | DPO3032 | DPO3034 | DPO3052 | DPO3054 |
|---|---|---|---|---|---|---|
| Input Channels | 2 | 4 | 2 | 4 | 2 | 4 |
| Analog Bandwidth (–3dB) | 100 MHz | 100 MHz | 300 MHz | 300 MHz | 500 MHz | 500 MHz |
| Calculated Rise Time 5 mV/div (typical) | 3.5 ns | 3.5 ns | 1.17 ns | 1.17 ns | 700 ps | 700 ps |
| Hardware Bandwidth Limits | 20 MHz or 150 MHz | | | | | |
| Input Coupling | AC, DC, GND | | | | | |
| Input Impedance | 1 MΩ ±1%, 75 Ω ±1%, 50 Ω ±1% | | | | | |
| Input Sensitivity Range, 1 MΩ | 1 mV/div to 10 V/div | | | | | |
| Input Sensitivity Range, 75 Ω, 50 Ω | 1 mV/div to 1 V/div | | | | | |
| Vertical Resolution | 8 bits (11 bits with Hi-Res) | | | | | |
| Max Input Voltage, 1 MΩ | 300 $V_{RMS}$ with peaks ≤±450 V | | | | | |
| Max Input Voltage, 75 Ω, 50 Ω | 5 $V_{RMS}$ with peaks ≤± 20 V | | | | | |
| DC Gain Accuracy | ±1.5% with offset set to 0 V | | | | | |
| **Offset Range** | | **1 MΩ** | | | **50 Ω, 75 Ω** | |
| 1 mV/div to 99.5 mV/div | | ±1 V | | | ±1 V | |
| 100 mV/div to 995 mV/div | | ±10 V | | | ±5 V | |
| 1 V/div | | ±100 V | | | ±5 V | |
| 1.01 V/div to 10 V/div | | ±100 V | | | NA | |
| Channel-to-Channel Isolation (Any Two Channels at Equal Vertical Scale) | ≥ 100:1 at ≤100 MHz and ≥ 30:1 at > 100 MHz up to the rated BW | | | | | |

# Agilent 34410A and 34411A Multimeters

Setting the Standard for Next Generation Benchtop and System Testing

Product Overview

# Agilent 34410A/11A
# 6 ½ Digit Multimeter

*(includes the L4411A 1U DMM)*

## User's Guide

Agilent Technologies


**152 Pages**


## **Accuracy Specifications** ± (% of reading + % of range)[1]

| Function | Range[3] | Frequency, Test Current or Burden Voltage | 24 Hour[2] Tcal ±1°C | 90 Day Tcal ±5°C | 1 Year Tcal ±5°C | Temperature Coefficient/°C 0°C to (Tcal -5°C) (Tcal +5°C) to 55°C |
|---|---|---|---|---|---|---|
| **DC Voltage** | 100.0000 mV | | 0.0030 + 0.0030 | 0.0040 + 0.0035 | 0.0050 + 0.0035 | 0.0005 + 0.0005 |
| | 1.000000 V | | 0.0020 + 0.0006 | 0.0030 + 0.0007 | 0.0035 + 0.0007 | 0.0005 + 0.0001 |
| | **10.00000 V** | | **0.0015 + 0.0004** | **0.0020 + 0.0005** | **0.0030 + 0.0005** | **0.0005 + 0.0001** |
| | 100.0000 V | | 0.0020 + 0.0006 | 0.0035 + 0.0006 | 0.0040 + 0.0006 | 0.0005 + 0.0001 |
| | 1000.000 V[4] | | 0.0020 + 0.0006 | 0.0035 + 0.0006 | 0.0040 + 0.0006 | 0.0005 + 0.0001 |
| **True RMS AC Voltage**[5] | 100.0000 mV to 750.000 V | 3 Hz – 5 Hz | 0.50 + 0.02 | 0.50 + 0.03 | 0.50 + 0.03 | 0.010 + 0.003 |
| | | 5 Hz – 10 Hz | 0.10 + 0.02 | 0.10 + 0.03 | 0.10 + 0.03 | 0.008 + 0.003 |
| | | **10 Hz – 20 kHz** | **0.02 + 0.02** | **0.05 + 0.03** | **0.06 + 0.03** | **0.005 + 0.003** |
| | | 20 kHz – 50 kHz | 0.05 + 0.04 | 0.09 + 0.05 | 0.10 + 0.05 | 0.010 + 0.005 |
| | | 50 kHz – 100 kHz | 0.20 + 0.08 | 0.30 + 0.08 | 0.40 + 0.08 | 0.020 + 0.008 |
| | | 100 kHz – 300 kHz | 1.00 + 0.50 | 1.20 + 0.50 | 1.20 + 0.50 | 0.120 + 0.020 |
| **Resistance**[6] | 100.0000 Ω | 1 mA | 0.0030 + 0.0030 | 0.008 + 0.004 | 0.010 + 0.004 | 0.0006 + 0.0005 |
| | 1.000000 kΩ | 1 mA | 0.0020 + 0.0005 | 0.007 + 0.001 | 0.010 + 0.001 | 0.0006 + 0.0001 |
| | **10.00000 kΩ** | **100 µA** | **0.0020 + 0.0005** | **0.007 + 0.001** | **0.010 + 0.001** | **0.0006 + 0.0001** |
| | 100.0000 kΩ | 10 µA | 0.0020 + 0.0005 | 0.007 + 0.001 | 0.010 + 0.001 | 0.0006 + 0.0001 |
| | 1.000000 MΩ | 5 µA | 0.0020 + 0.0010 | 0.010 + 0.001 | 0.012 + 0.001 | 0.0010 + 0.0002 |
| | 10.00000 MΩ | 500 nA | 0.0100 + 0.0010 | 0.030 + 0.001 | 0.040 + 0.001 | 0.0030 + 0.0004 |
| | 100.0000 MΩ | 500 nA \|\| 10 MΩ | 0.200 + 0.001 | 0.600 + 0.001 | 0.800 + 0.001 | 0.1000 + 0.0001 |
| | 1.000000 GΩ | 500 nA \|\| 10 MΩ | 2.000 + 0.001 | 6.000 + 0.001 | 8.000 + 0.001 | 1.0000 + 0.0001 |
| **DC Current** | 100.0000 µA | < 0.03 V | 0.010 + 0.020 | 0.040 + 0.025 | 0.050 + 0.025 | 0.0020 + 0.0030 |
| | **1.000000 mA** | **< 0.3 V** | **0.007 + 0.006** | **0.030 + 0.006** | **0.050 + 0.006** | **0.0020 + 0.0005** |
| | 10.00000 mA | < 0.03 V | 0.007 + 0.020 | 0.030 + 0.020 | 0.050 + 0.020 | 0.0020 + 0.0020 |
| | 100.0000 mA | < 0.3 V | 0.010 + 0.004 | 0.030 + 0.005 | 0.050 + 0.005 | 0.0020 + 0.0005 |
| | 1.000000 A | < 0.8 V | 0.050 + 0.006 | 0.080 + 0.010 | 0.100 + 0.010 | 0.0050 + 0.0010 |
| | 3.000000 A | < 2.0 V | 0.100 + 0.020 | 0.120 + 0.020 | 0.150 + 0.020 | 0.0050 + 0.0020 |


Agilent
-25.00V
1.000A
ADJUST
FUNCTION

Part Number: E3631-90002
October 2007.

For Safety information, Warranties, and Regulatory information, see the pages behind the Index.



# Agilent E3631A Triple Output DC Power Supply

168 Pages


When is the voltage reading on the Signal Generator Accurate?

**Almost Never !**

When it is reasonably close, (i.e. when not affected by the effects of the output impedance on the actual output) how does the accuracy compare with that of the scope or the digital multimeter measuring the same voltage?

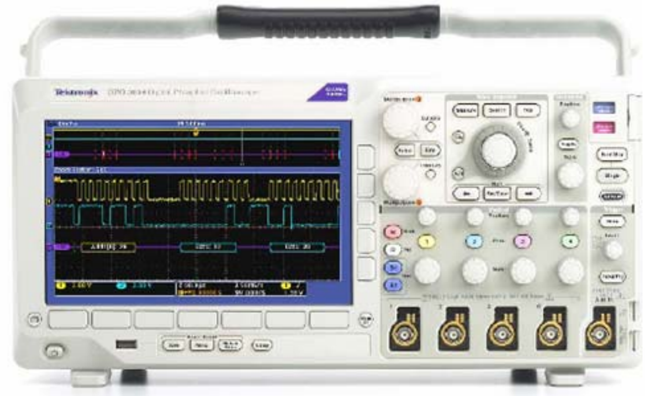
**Two to three orders of magnitude worse !**

# Agilent 33220A 20 MHz Function/Arbitrary Waveform Generator

| WAVEFORMS | |
|---|---|
| Standard | Sine, Square, Ramp, Triangle, Pulse, Noise, DC |
| Built-in arbitrary | Exponential rise, Exponential fall, Negative ramp, Sin(x)/x, Cardiac |

| WAVEFORM CHARACTERISTICS | | |
|---|---|---|
| **Sine** | | |
| Frequency Range | 1 μHz to 20 MHz | |
| Amplitude Flatness[1],[2] | (relative to 1 kHz) | |
| | < 100 kHz | 0.1 dB |
| | 100 kHz to 5 MHz | 0.15 dB |
| | 5 MHz to 20 MHz | 0.3 dB |
| Harmonic distortion[2],[3] | $< 1\ V_{PP}$ | $\geq 1\ V_{PP}$ |
| DC to 20 kHz | -70 dBc | -70 dBc |
| 20 kHz to 100 kHz | -65 dBc | -60 dBc |
| 100 kHz to 1 MHz | -50 dBc | -45 dBc |
| 1 MHz to 20 MHz | -40 dBc | -35 dBc |
| Total harmonic distortion[2][3] | | |
| DC to 20 kHz | 0.04% | |
| Spurious (non-harmonic)[2][4] | | |
| DC to 1 MHz | -70 dBc | |
| 1 MHz to 20 MHz | -70 dBc + 6 dB/octave | |
| Phase noise (10 kHz offset) | -115 dBc / Hz, typical | |

| COMMON CHARACTERISTICS | |
|---|---|
| **Frequency** | |
| Resolution | 1 μHz |
| **Amplitude** | |
| Range | $10\ mV_{PP}$ to $10\ V_{PP}$ into 50Ω<br>$20\ mV_{PP}$ to $20\ V_{PP}$ into open circuit |
| Accuracy[1][2] (at 1 kHz) | ± 1% of setting ± $1\ mV_{PP}$ |
| Units | $V_{PP}$, $V_{rms}$, dBm |
| Resolution | 4 digits |
| **DC Offset** | |
| Range (peak AC + DC) | ± 5 V into 50Ω<br>± 10 V into open circuit |
| Accuracy[1][2] | ± 2% of offset setting<br>± 0.5% of amplitude ± 2 mV |
| Resolution | 4 digits |
| **Main Output** | |
| Impedance | 50 Ω typical |
| Isolation | 42 Vpk maximum to earth |
| Protection | Short-circuit protected, overload automatically disables main output |
| **Internal Frequency Reference** | |
| Accuracy[5] | ± 10 ppm in 90 days<br>± 20 ppm in 1 year |
| **External Frequency Reference (Option 001)** | |



## ▸ Characteristics

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| Calculated Rise Time 5 mV/div (typical) | 3.5 ns | 3.5 ns | 1.17 ns | 1.17 ns | 700 ps | 700 ps |
| Hardware Bandwidth Limits | 20 MHz or 150 MHz | | | | | |
| Input Coupling | AC, DC, GND | | | | | |
| Input Impedance | 1 MΩ ±1%, 75 Ω ±1%, 50 Ω ±1% | | | | | |
| Input Sensitivity Range, 1 MΩ | 1 mV/div to 10 V/div | | | | | |
| Input Sensitivity Range, 75 Ω, 50 Ω | 1 mV/div to 1 V/div | | | | | |
| Vertical Resolution | 8 bits (11 bits with Hi-Res) | | | | | |
| Max Input Voltage, 1 MΩ | 300 $V_{RMS}$ with peaks ≤±400 V | | | | | |
| Max Input Voltage, 75 Ω, 50 Ω | 5 $V_{RMS}$ with peaks ≤±20 V | | | | | |
| DC Gain Accuracy | ±1.5% with offset set to 0 V | | | | | |
| **Offset Range** | | **1 MΩ** | | | **50 Ω, 75 Ω** | |
| 1 mV/div to 99.5 mV/div | | ±1 V | | | ±1 V | |
| 100 mV/div to 995 mV/div | | ±10 V | | | ±5 V | |
| 1 V/div | | ±100 V | | | ±5 V | |
| 1.01 V/div to 10 V/div | | ±100 V | | | NA | |
| Channel-to-Channel Isolation (Any Two Channels at Equal Vertical Scale) | ≥ 100:1 at ≤100 MHz and ≥ 30:1 at > 100 MHz up to the rated BW | | | | | |


## Accuracy Specifications ± (% of reading + % of range)[1]

| Function | Range[3] | Frequency, Test Current or Burden Voltage | 24 Hour[2] Tcal ±1°C | 90 Day Tcal ±5°C | 1 Year Tcal ±5°C | Temperature Coefficient/°C 0°C to (Tcal -5°C) (Tcal +5°C) to 55°C |
|---|---|---|---|---|---|---|
| **DC Voltage** | 100.0000 mV | | 0.0030 + 0.0030 | 0.0040 + 0.0035 | 0.0050 + 0.0035 | 0.0005 + 0.000[illegible] |
| | 1.000000 V | | 0.0020 + 0.0006 | 0.0030 + 0.0007 | 0.0035 + 0.0007 | 0.0005 + 0.000[illegible] |
| | **10.00000 V** | | **0.0015 + 0.0004** | **0.0020 + 0.0005** | **0.0030 + 0.0005** | **0.0005 + 0.000[illegible]** |
| | 100.0000 V | | 0.0020 + 0.0006 | 0.0035 + 0.0006 | 0.0040 + 0.0006 | 0.0005 + 0.000[illegible] |
| | 1000.000 V[4] | | 0.0020 + 0.0006 | 0.0035 + 0.0006 | 0.0040 + 0.0006 | 0.0005 + 0.0001 |
| **True RMS AC Voltage**[5] | 100.0000 mV to 750.000 V | 3 Hz – 5 Hz | 0.50 + 0.02 | 0.50 + 0.03 | 0.50 + 0.03 | 0.010 + 0.003 |
| | | 5 Hz – 10 Hz | 0.10 + 0.02 | 0.10 + 0.03 | 0.10 + 0.03 | 0.008 + 0.003 |
| | | **10 Hz – 20 kHz** | **0.02 + 0.02** | **0.05 + 0.03** | **0.06 + 0.03** | **0.005 + 0.003** |
| | | 20 kHz – 50 kHz | 0.05 + 0.04 | 0.09 + 0.05 | 0.10 + 0.05 | 0.010 + 0.005 |
| | | 50 kHz – 100 kHz | 0.20 + 0.08 | 0.30 + 0.08 | 0.40 + 0.08 | 0.020 + 0.008 |
| | | 100 kHz – 300 kHz | 1.00 + 0.50 | 1.20 + 0.50 | 1.20 + 0.50 | 0.120 + 0.020 |
| **Resistance**[6] | 100.0000 Ω | 1 mA | 0.0030 + 0.0030 | 0.008 + 0.004 | 0.010 + 0.004 | 0.0006 + 0.0005 |
| | 1.000000 kΩ | 1 mA | 0.0020 + 0.0005 | 0.007 + 0.001 | 0.010 + 0.001 | 0.0006 + 0.0001 |
| | **10.00000 kΩ** | **100 μA** | **0.0020 + 0.0005** | **0.007 + 0.001** | **0.010 + 0.001** | **0.0006 + 0.0001** |
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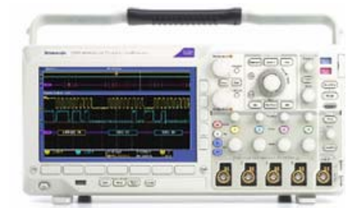
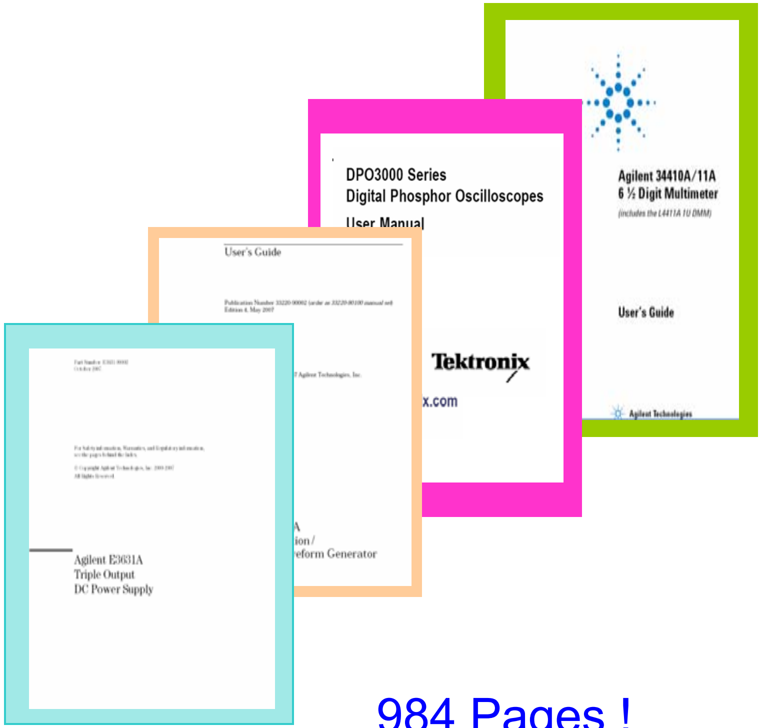
| | 10.00000 MΩ | 500 nA | 0.0100 + 0.0010 | 0.030 + 0.001 | 0.040 + 0.001 | 0.0030 + 0.0004 |
| | 100.0000 MΩ | 500 nA \|\| 10 MΩ | 0.200 + 0.001 | 0.600 + 0.001 | 0.800 + 0.001 | 0.1000 + 0.0001 |
| | 1.000000 GΩ | 500 nA \|\| 10 MΩ | 2.000 + 0.001 | 6.000 + 0.001 | 8.000 + 0.001 | 1.0000 + 0.0001 |
| **DC Current** | 100.0000 μA | < 0.03 V | 0.010 + 0.020 | 0.040 + 0.025 | 0.050 + 0.025 | 0.0020 + 0.0030 |
| | **1.000000 mA** | **< 0.3 V** | **0.007 + 0.006** | **0.030 + 0.006** | **0.050 + 0.006** | **0.0020 + 0.0005** |
| | 10.00000 mA | < 0.03 V | 0.007 + 0.020 | 0.030 + 0.020 | 0.050 + 0.020 | 0.0020 + 0.0020 |
| | 100.0000 mA | < 0.3 V | 0.010 + 0.004 | 0.030 + 0.005 | 0.050 + 0.005 | 0.0020 + 0.0005 |
| | 1.000000 A | < 0.8 V | 0.050 + 0.006 | 0.080 + 0.010 | 0.100 + 0.010 | 0.0050 + 0.0010 |
| | 3.000000 A | < 2.0 V | 0.100 + 0.020 | 0.120 + 0.020 | 0.150 + 0.020 | 0.0050 + 0.0020 |

# Test Equipment in the EE 230 Laboratory






984 Pages !

- The documentation for the operation of this equipment is extensive
- Critical that user always know what equipment is doing
- Consult the users manuals and specifications whenever unsure

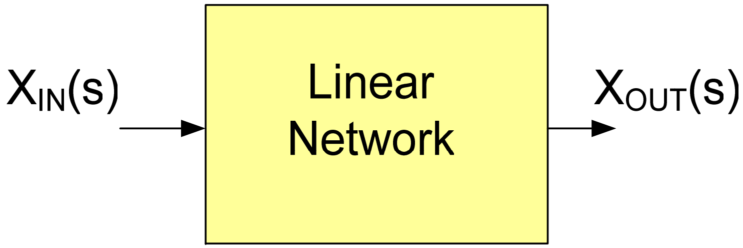
# Whats inside/on this equipment?

- Computer (except maybe dc power supply)
- Some analog circuitry
- Software
- Knobs/Buttons
- Computer Interface

**Why is this not the standard interface with a computer?**

Review from Last Time

# Properties of Linear Networks




frequency domain

$$\frac{X_{OUT}(s)}{X_{IN}(s)} = T(s)$$

$T(s)$ is termed the transfer function

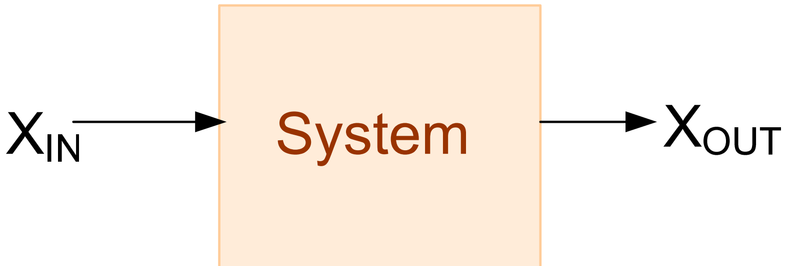
This is often termed the “s-domain” or “Laplace-domain” representation

$$T(s)\Big|_{s=j\omega} = T_P(j\omega)$$

**Will discuss the frequency domain representations and the more general concept of transfer functions in more detail later**

Review from Last Time

# Distortion




A system has Harmonic Distortion (often just termed “Distortion”) if when a pure sinusoidal input is applied, the Fourier Series representation of the output contains one or more terms at frequencies different than the input frequency

A linear system has Frequency Distortion if for any two sinusoidal inputs of magnitude $X_1$ and $X_2$, the ratio of the corresponding sinusoidal outputs is not equal to $X_1/X_2$.

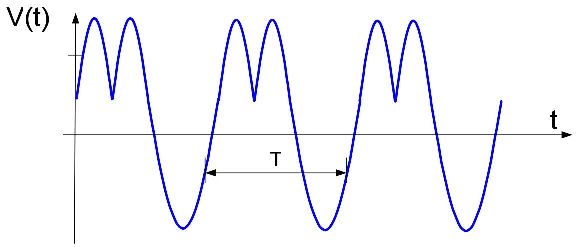
Harmonic distortion is characterized by several different metrics including the Total Harmonic Distortion, Spurious Free Dynamic Range (SFDR)

Frequency distortion is characterized by the transfer function, T(s), of the system

Review from Last Time

# Total Harmonic Distortion

$$P_{AVG} = \frac{1}{T}\int_{t_1}^{t_1+T} f^2(t)\,dt$$




$$f(t) = \sum_{k=1}^{\infty} A_k \sin(k\omega t+\theta_k)$$

It can be shown that

$$P_{AVG} = \frac{\sum_{k=1}^{\infty} A_k^2}{2}$$

Define $P_1$ to be the power in the fundamental

$$P_1 = \frac{A_1^2}{2}$$

$$P_{Harmonics} = \frac{\sum_{k=2}^{\infty} A_k^2}{2}$$

$$THD = \frac{P_{HARMONICS}}{P_1}$$

$$THD = \frac{\sum_{k=2}^{\infty} A_k^2}{A_1^2}$$

THD often expressed in dB or in %

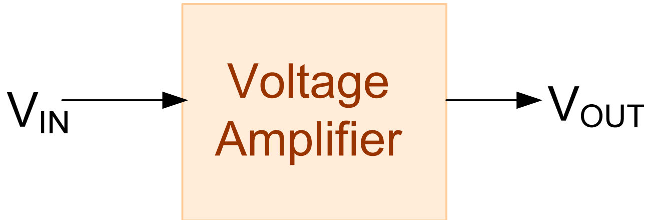
$$THD_{dB} = 10\log_{10}(THD)$$

Can also be expressed relative to signal instead of power

# Amplifiers:

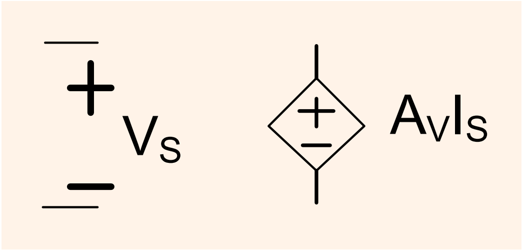
Review from Last Time

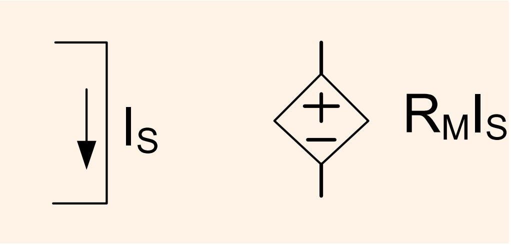
Amplifiers are circuits that scale a signal by a constant amount

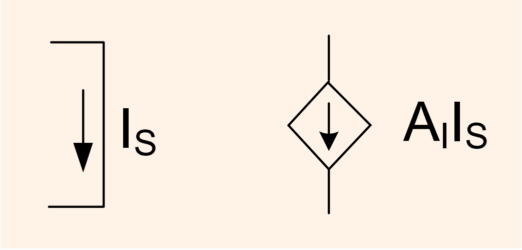


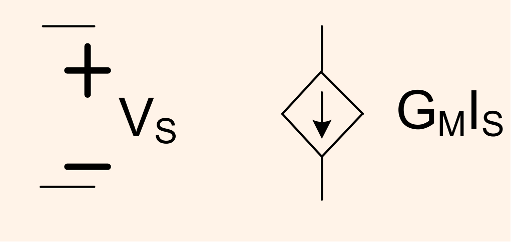

Ideally $V_{OUT}=AV_{IN}$ where A is a constant (termed the gain)

The dependent sources discussed in EE 201 are amplifiers

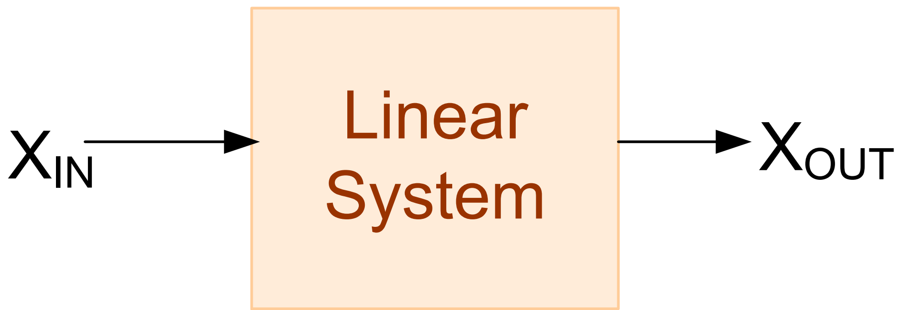


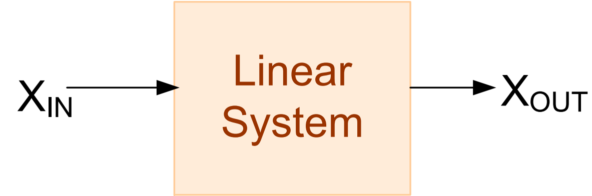
# Amplifiers, Frequency Response, and Transfer Functions



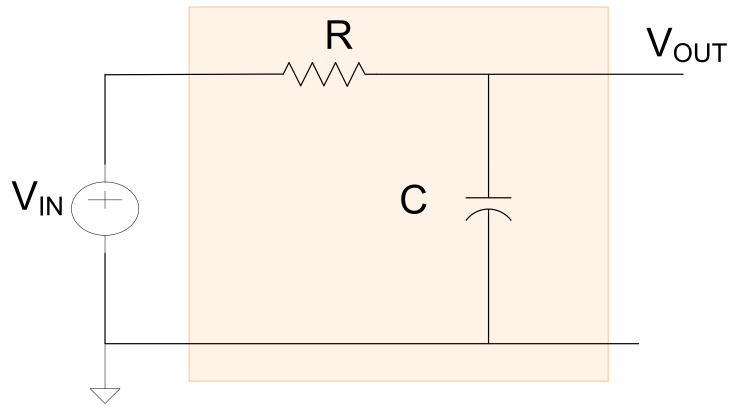

The frequency dependent gain of a linear circuit or system is often termed the transfer function

Sometimes linear circuits are termed “Amplifiers” or “Filters” when some specific properties of the relationship between the input and output are of particular interest

# Example:



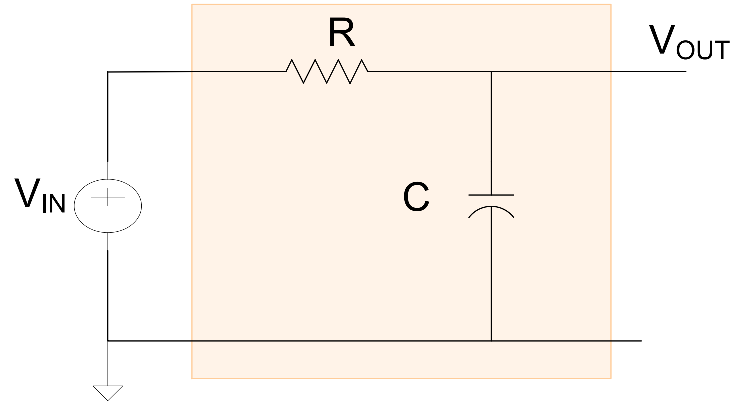

Will go through the mechanics first, then formalize the concepts



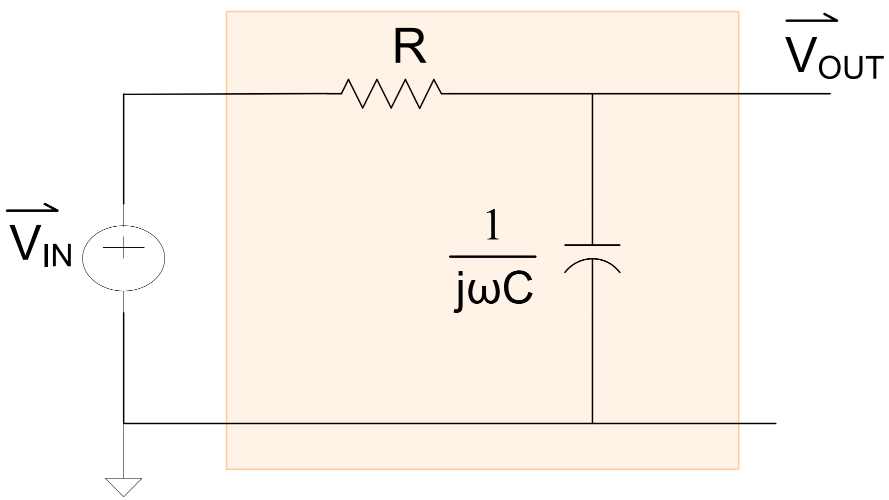

- Obtain the phasor-domain transfer function
- Obtain the s-domain transfer function
- Plot the magnitude of the transfer function
- Plot the phase of the transfer function
- Obtain the sinusoidal steady state response if $V_{IN}=V_M\sin(2\pi\ f\ t)$
- Do a time-domain analysis of this circuit

# Example:

- Obtain the phasor-domain transfer function




Phasor-Domain Circuit



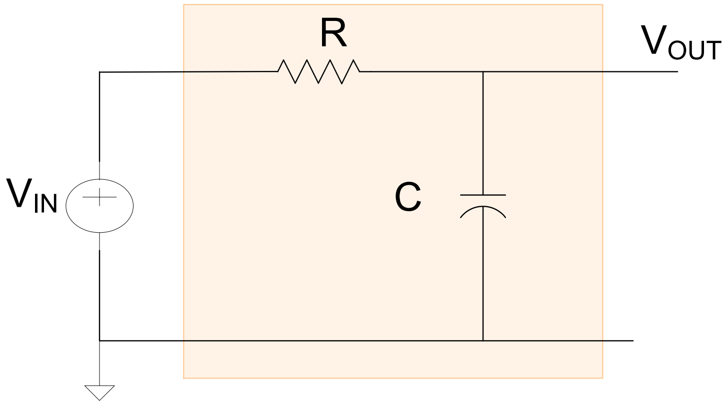

$$\vec{V}_{OUT} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \vec{V}_{IN}$$

$$\vec{V}_{OUT} = \frac{1}{1 + j\omega RC} \vec{V}_{IN}$$

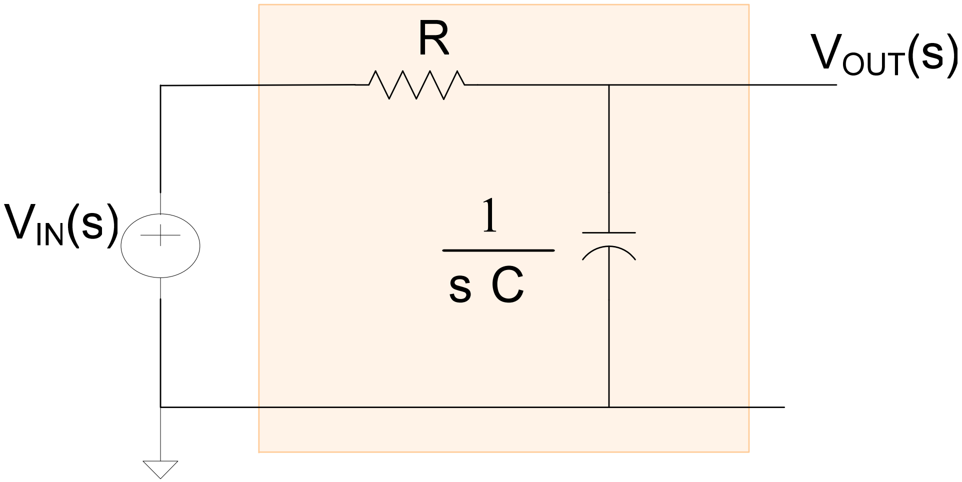
$$T_P(j\omega) = \frac{\vec{V}_{OUT}(j\omega)}{\vec{V}_{IN}(j\omega)} = \frac{1}{1 + j\omega RC}$$

# Example:

- Obtain the s-domain transfer function




s-Domain Circuit

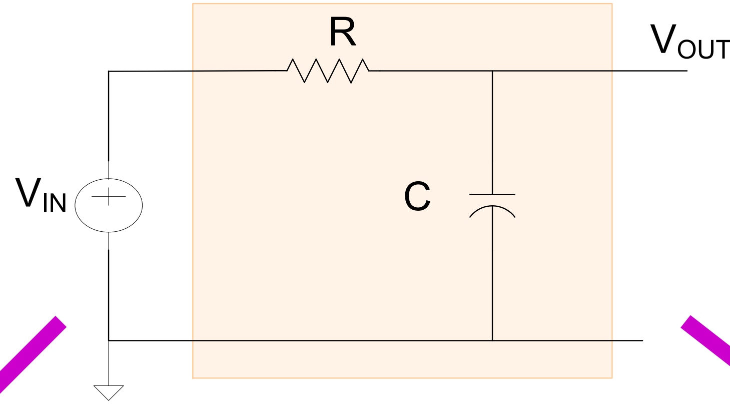


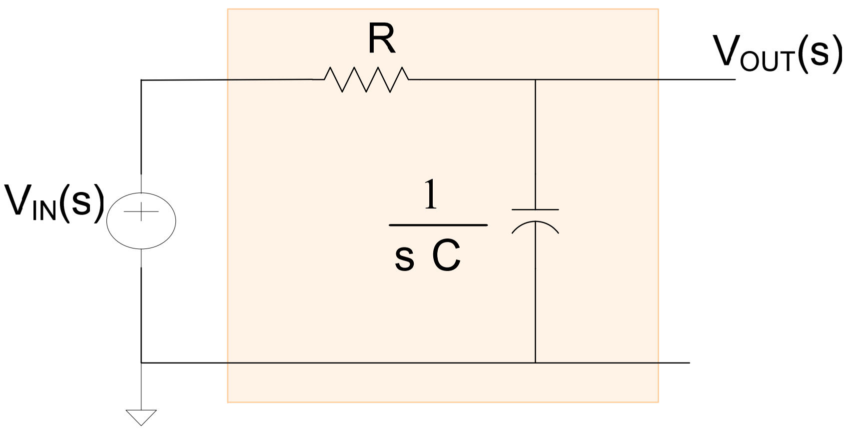

$$V_{OUT}(s) = \frac{\frac{1}{sC}}{R+\frac{1}{sC}} V_{IN}(s)$$

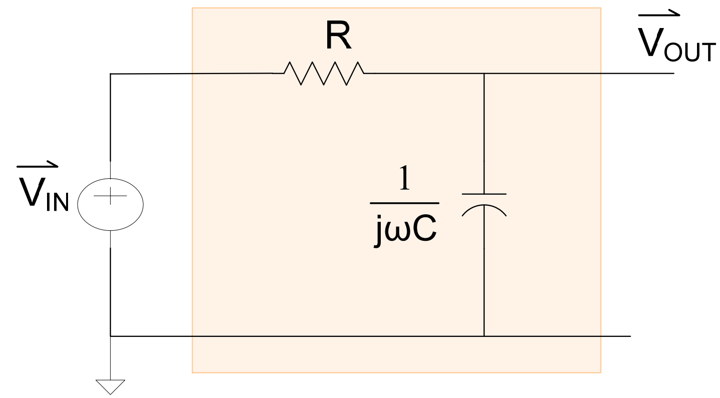
$$V_{OUT}(s) = \frac{1}{1+sRC} V_{IN}(s)$$

$$T(s) = \frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{1}{1+sRC}$$

# Example:




$$T(s) = \frac{1}{1+sRC}$$

$$T_P(j\omega) = \frac{1}{1+j\omega RC}$$

Observe:

$$T(s)\Big|_{s=j\omega} = \frac{1}{1+j\omega RC}$$

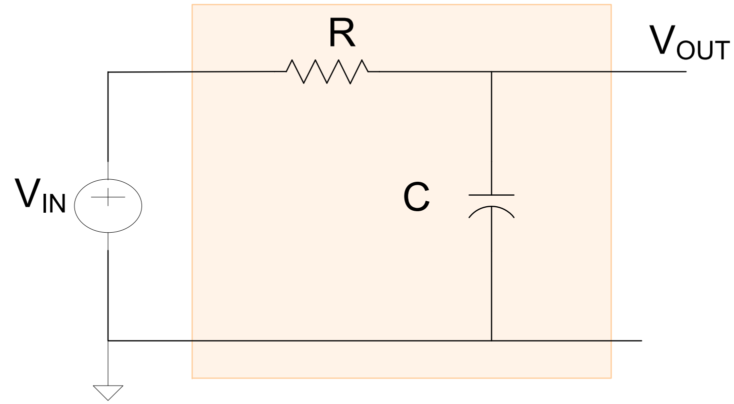
Observe:

$$T(s)\Big|_{s=j\omega} = T_P(j\omega)$$

This property holds for any linear system !

# Example:

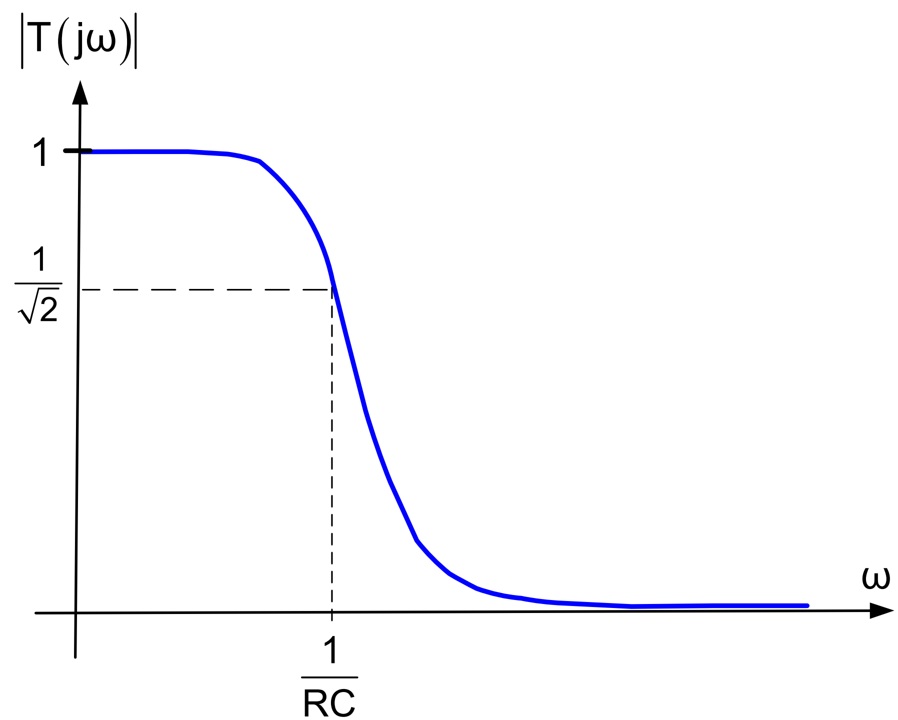
- Plot the transfer function magnitude



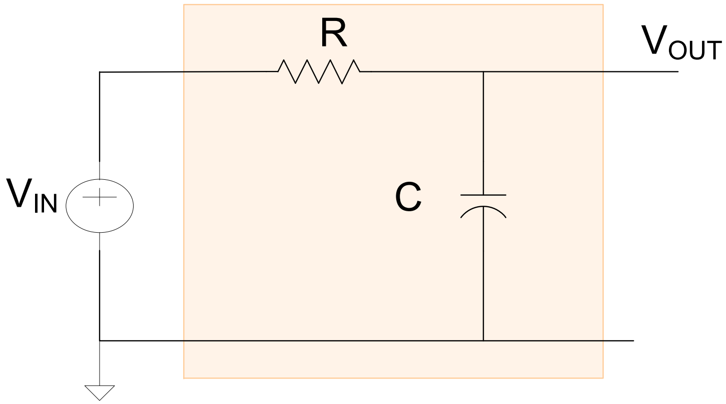

$$T(s) = \frac{1}{1+sRC}$$

$$T(j\omega) = \frac{1}{1+j\omega RC}$$

$$\left|T(j\omega)\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$



# Example:

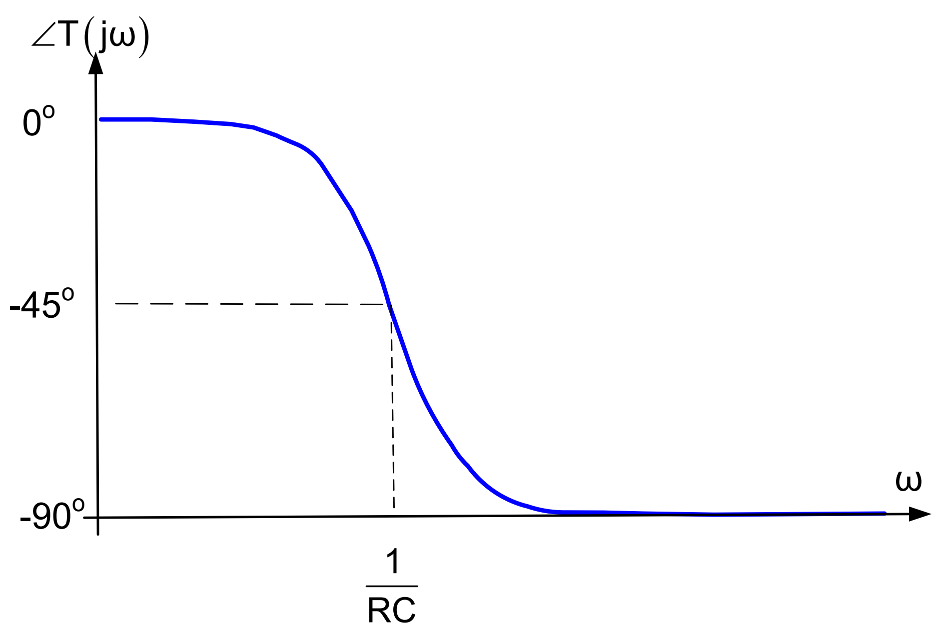



- Plot the phase of the transfer function

$$T(s) = \frac{1}{1+sRC}$$

$$T(j\omega) = \frac{1}{1+j\omega RC}$$

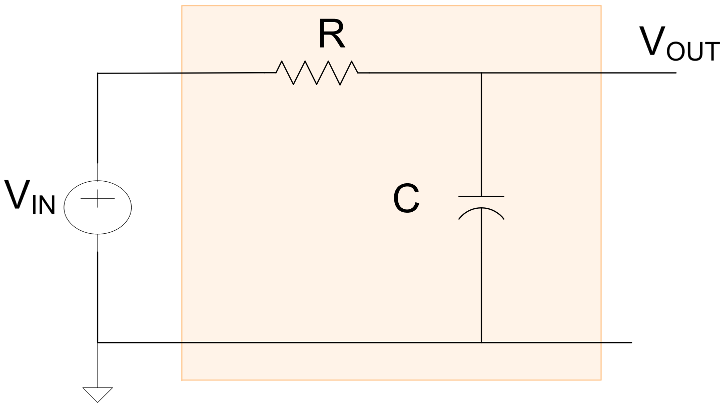
$$\angle T(j\omega) = -\tan^{-1}(\omega RC)$$



# Example:

- Obtain the sinusoidal steady-state response if

$$V_{IN}=V_M\sin(2\pi\ f\ t)$$




Need a theorem that expresses the sinusoidal steady-state response

# Key Theorem:

Theorem: The steady-state response of a linear network to a sinusoidal excitation of $V_{IN} = V_M \sin(\omega t + \gamma)$ is given by

$$V_{OUT}(t) = V_m \left|T(j\omega)\right| \sin\left(\omega t + \gamma + \angle T(j\omega)\right)$$

This is a very important theorem and is one of the major reasons phasor analysis was studied in EE 201

The sinusoidal steady state response is completely determined by $T(j\omega)$

The sinusoidal steady state response can be written by inspection from the

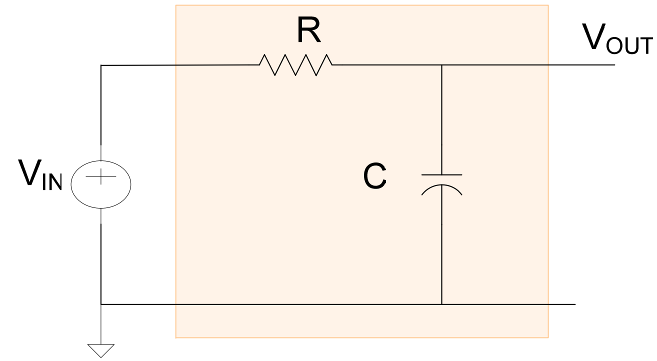
$\left|T(j\omega)\right|$ and $\angle T(j\omega)$ plots

$$T(s)\big|_{s=j\omega} = T_P(j\omega)$$

# Example:

- Obtain the sinusoidal steady-state response if

$V_{IN}=V_M \sin(2\pi f t)$




$$|T(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}} \qquad \angle T(j\omega) = -\tan^{-1}(\omega RC)$$

Thus, from the previous theorem with γ=0

$$V_{OUT}(t) = V_m |T(j\omega)| \sin(\omega t + \gamma + \angle T(j\omega))$$

$$V_{OUT}(t) = V_m \frac{1}{\sqrt{1+(\omega RC)^2}} \sin(\omega t - \tan^{-1}(\omega RC))$$

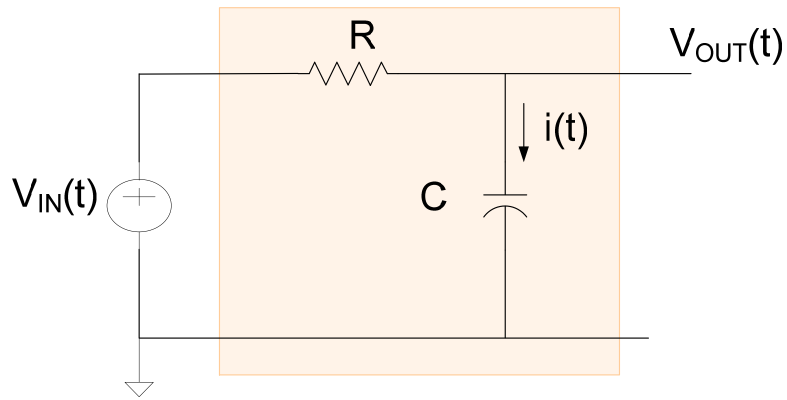
# Observations:

- Authors of current electronics textbooks do not talk about phasors or $T_P(j\omega)$
- This is consistent with the industry when discussing electronic circuits and systems
- The sinusoidal steady state response is of considerable concern in electronic circuits and is used extensively in the text for this cours
- Authors and industry use the concept of the transfer function T(s) when characterizing the frequency-dependent performance of linear circuits and systems

# Questions

- Why is $T(s)$ used instead of $T_P(j\omega)$ in the electronics field?
- What is $T(s)$?
- Why was $T_P(j\omega)$ emphasized in EE 201 instead of $T(s)$ for characterizing the frequency dependence of linear networks?

# Example:




- Do a time-domain analysis of this circuit

$$i(t)=\frac{V_{IN}(t)-V_{OUT}(t)}{R}$$

$$i(t)=C\frac{dV_{OUT}(t)}{dt}$$

$$V_{IN}(t)=V_M\sin(\omega t+\gamma)$$

Complete set of differential equations that can be solved to obtain $V_{OUT}(t)$

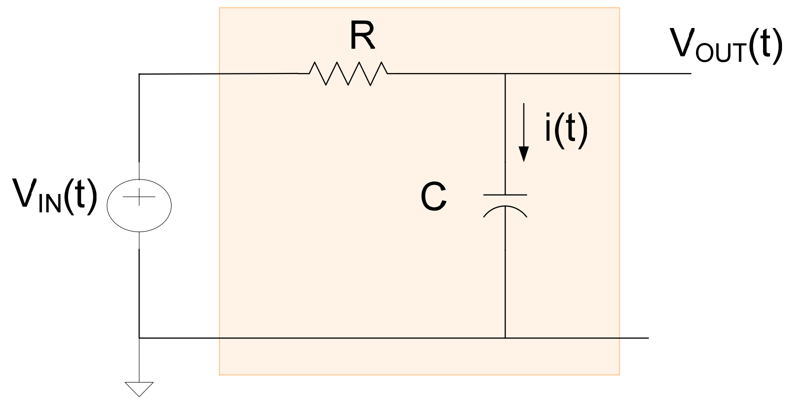
One way to solve this is to use Laplace Transforms

$$\mathcal{I}=sC\mathcal{V}_{OUT}$$

$$\mathcal{V}_{IN}-\mathcal{V}_{OUT}=\mathcal{I}R$$

$$\mathcal{V}_{IN}=V_M\frac{(\sin\gamma)s+\omega\cos\gamma}{s^2+\omega^2}$$

# Example:




- Do a time-domain analysis of this circuit

$$\left.\begin{aligned} \mathcal{I} &= sC\mathcal{V}_{OUT} \\ \mathcal{V}_{IN}-\mathcal{V}_{OUT} &= \mathcal{I}R \\ \mathcal{V}_{IN} &= V_M \frac{(\sin\gamma)s+\omega\cos\gamma}{s^2+\omega^2} \end{aligned}\right\}$$

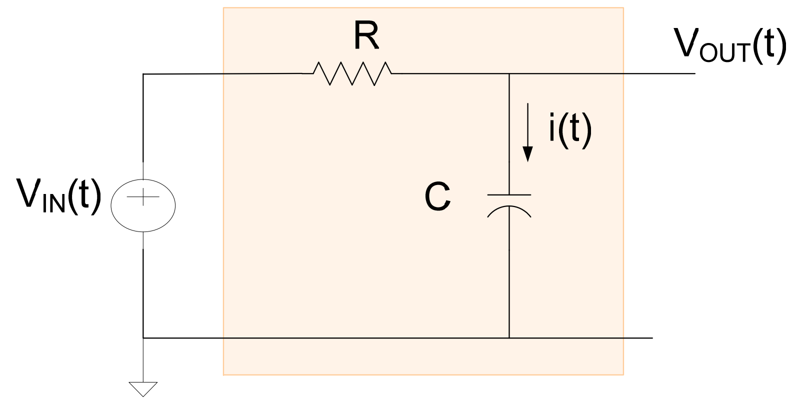
With some manipulations, can get expression for $\mathcal{V}_{OUT}$

$$\mathcal{V}_{OUT} = \left[V_M \frac{(\sin\gamma)s+\omega\cos\gamma}{s^2+\omega^2}\right]\left(\frac{1}{1+sRC}\right)$$

With some more manipulations, we can take inverse Laplace transform to get

$$\tilde{V}_{OUT}(t) = \left[V_M \frac{\sin\gamma}{(RC)^2}\left(1-\frac{\omega RC}{\tan\gamma}\right)e^{-t/RC}\right] + \left[V_M\left(\frac{1}{\sqrt{1+(\omega RC)^2}}\right)\sin\left(\omega t + \gamma - \tan^{-1}(\omega RC)\right)\right]$$

# Example:




- Do a time-domain analysis of this circuit

$$\tilde{V}_{OUT}(t)=\left[V_M\frac{\sin\gamma}{(RC)^2}\left(1-\frac{\omega RC}{\tan\gamma}\right)e^{-t/RC}\right]+\left[V_M\left(\frac{1}{\sqrt{1+(\omega RC)^2}}\right)\sin\left(\omega t+\gamma-\tan^{-1}(\omega RC)\right)\right]$$

Neglecting the natural response to obtain the sinusoidal steady state response, we obtain (with γ = 0)

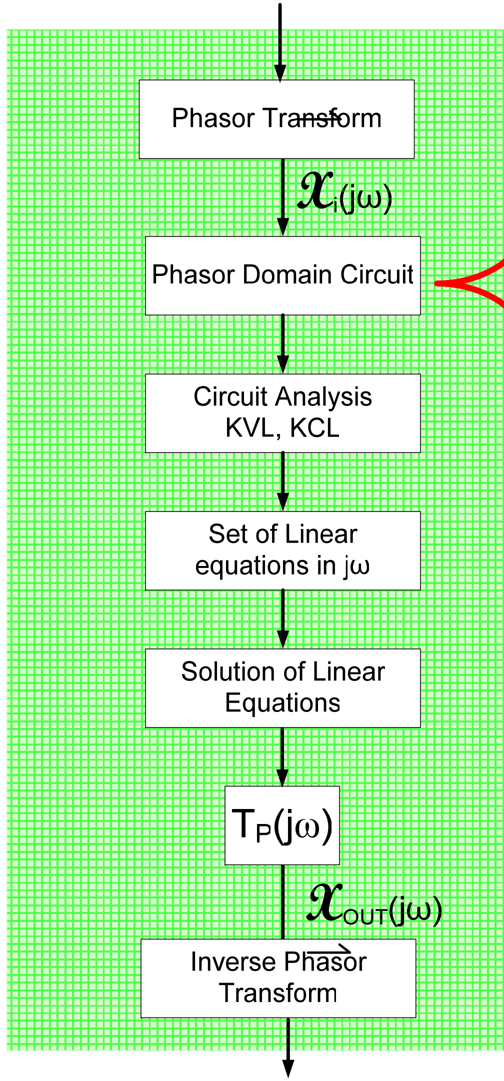
$$V_{OUT}(t)=V_M\left(\frac{1}{\sqrt{1+(\omega RC)^2}}\right)\sin\left(\omega t-\tan^{-1}(\omega RC)\right)$$

Note this is the same response as was obtained with the two previous solutons

# Formalization of sinusoidal steady-state analysis

phasor-domain approach

$X_i(t) = X_M \sin(\omega t + \theta)$




$X_{OUT}(t)$

$L \rightarrow j\omega L$

$C \rightarrow \frac{1}{j\omega C}$

All other components unchanged

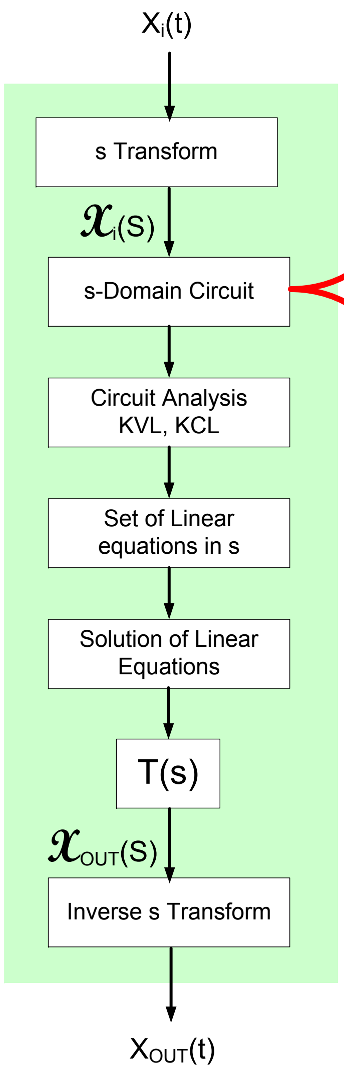
$\mathcal{X}_{OUT} = T_P(j\omega)\mathcal{X}_{IN}$ (this equation not needed)

$X_{OUT}(t) = X_M |T(j\omega)| \sin(\omega t + \theta + \angle T(j\omega))$

# Formalization of sinusoidal steady-state analysis

s-domain approach

$X_{IN}(t) = X_M \sin(\omega t + \theta)$



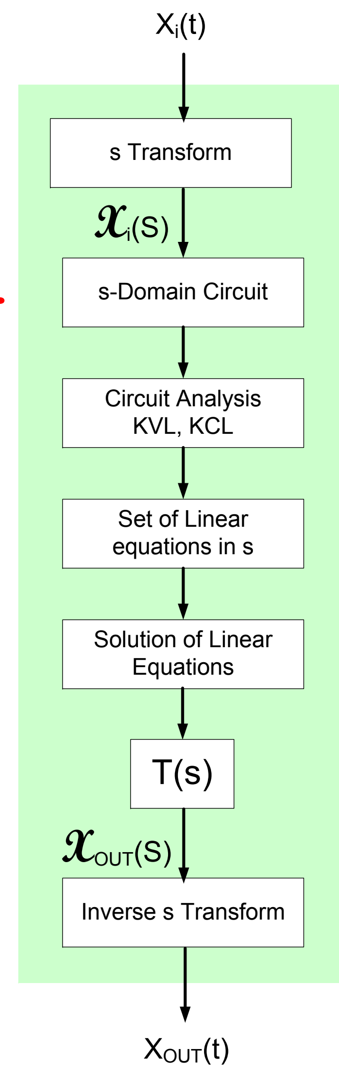

$L \rightarrow sL$

$C \rightarrow \frac{1}{sC}$

All other components unchanged

$$\mathcal{X}_{OUT} = T_P(j\omega)\,\mathcal{X}_{IN}$$

(this equation not needed)

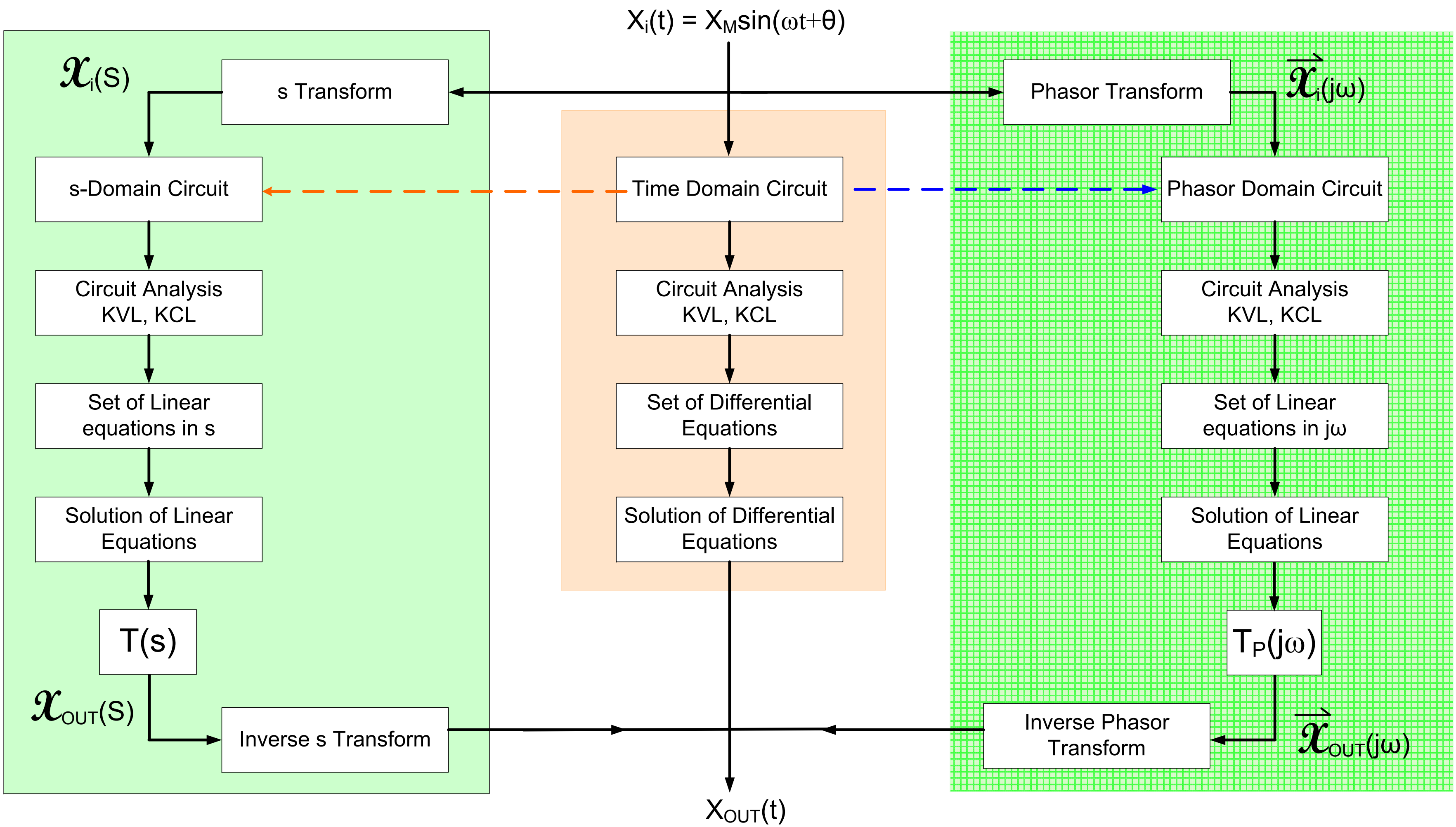



$X_{OUT}(t) = X_M \left|T(j\omega)\right| \sin\left(\omega t + \theta + \angle T(j\omega)\right)$

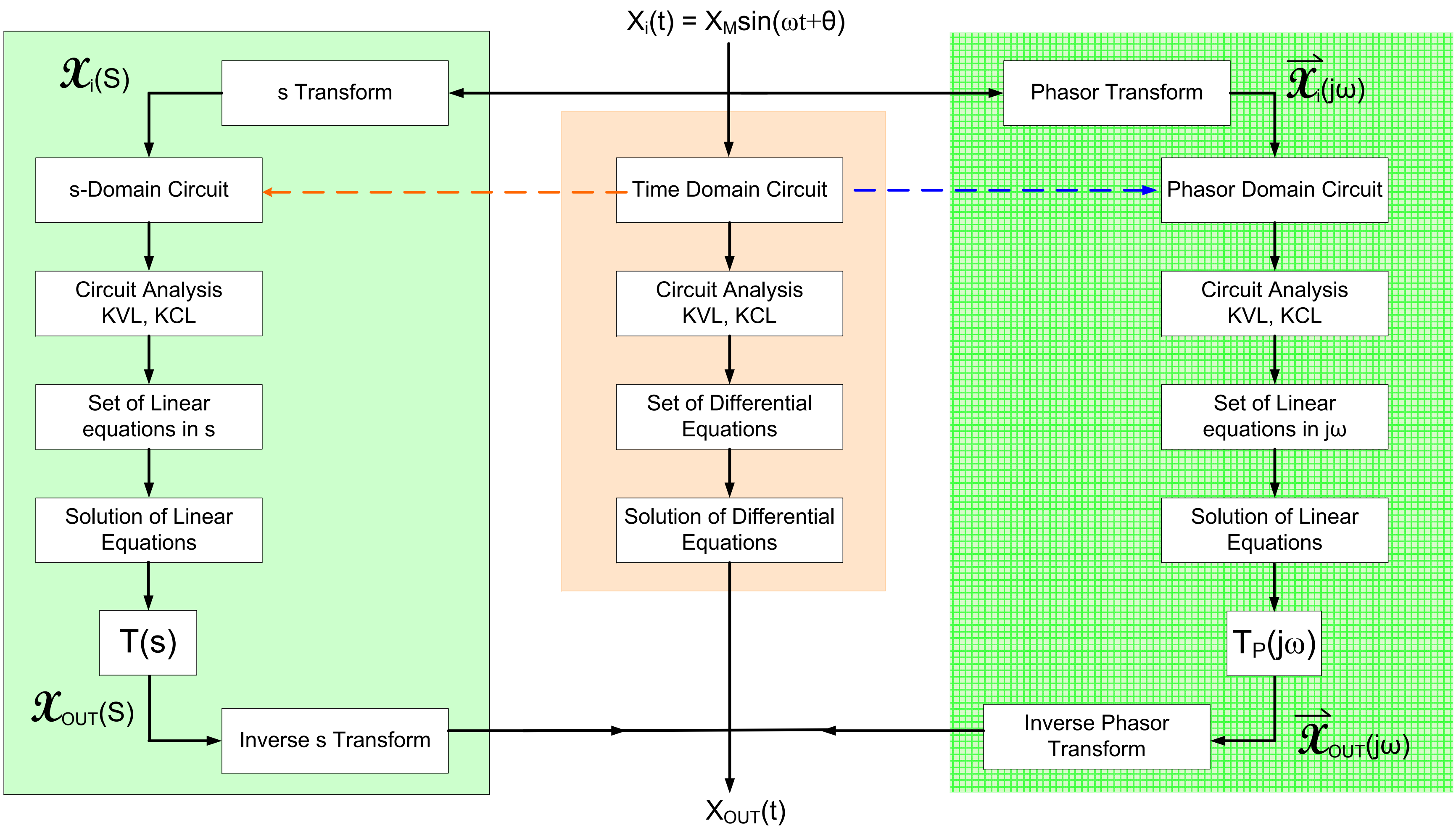
general

Sinusoidal steady state

# Formalization of sinusoidal steady-state analysis



# Which of the methods is most widely used?




s-domain analysis almost totally dominates the electronics fields and most systems fields

# Formalization of sinusoidal steady-state analysis - Summary

## s-domain The Preferred Approach

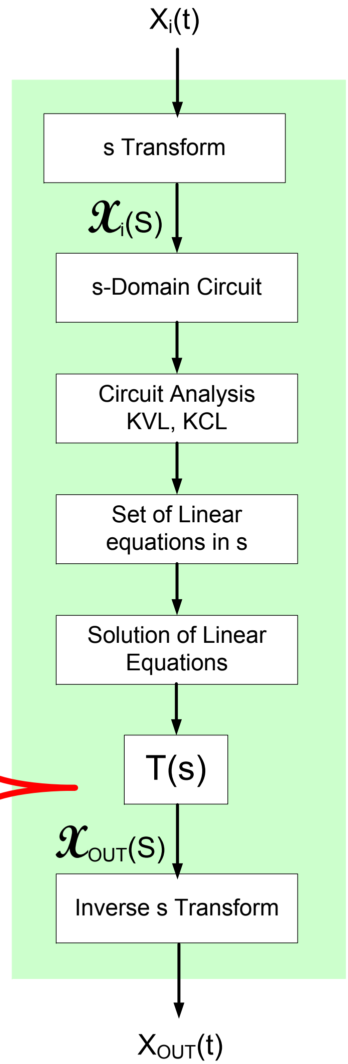
$$X_{IN}(t) = X_M \sin(\omega t + \theta)$$

$$L \rightarrow sL$$

$$C \rightarrow \frac{1}{sC}$$

All other components unchanged

$$X_{OUT}(t) = X_M \left|T(j\omega)\right| \sin\left(\omega t + \theta + \angle T(j\omega)\right)$$

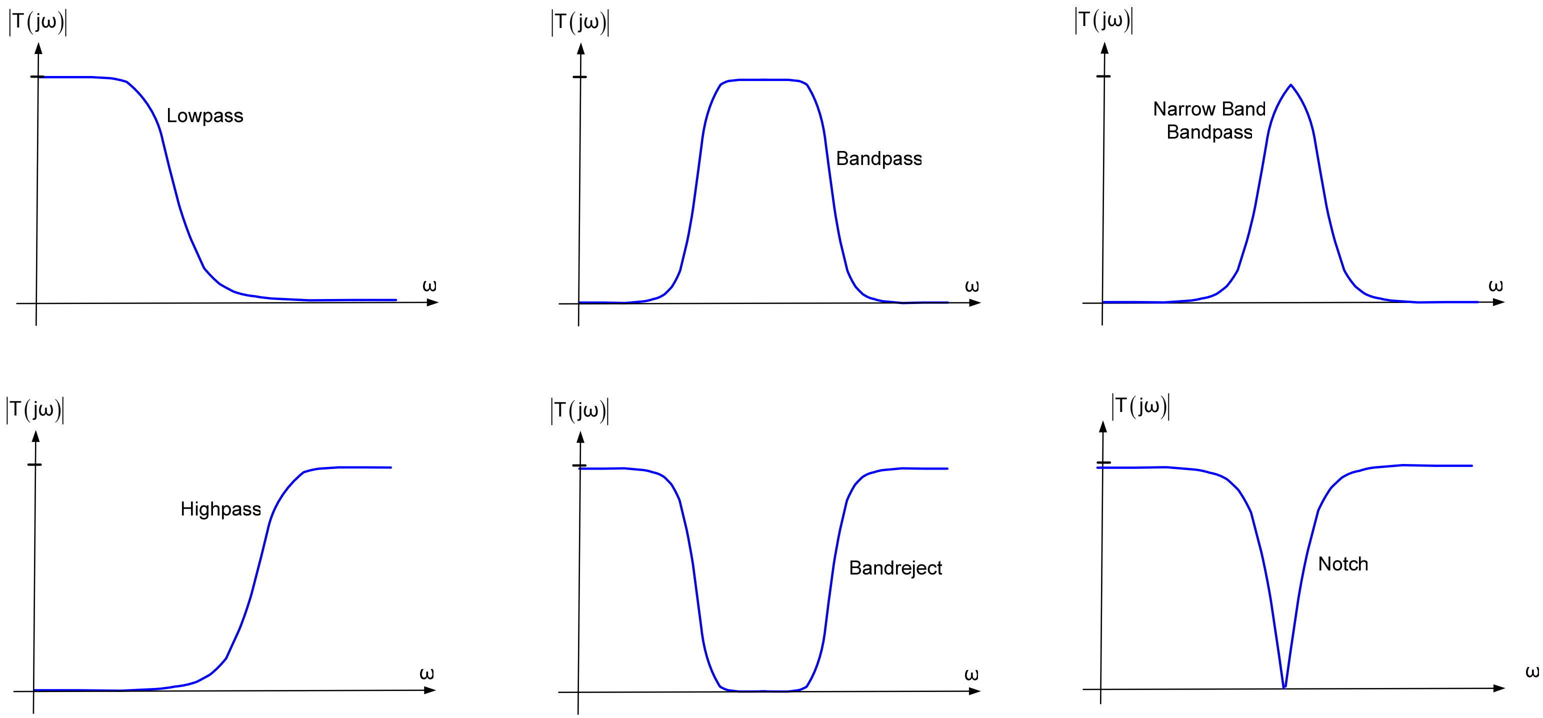


# Filters:

A filter is an amplifier that ideally has a frequency dependent gain

Simply a different name for an  amplifier that typically has an ideal magnitude or phase response that is not flat

Some standard filter responses with accepted nomenclature



# Summary of frequency response appears on posted notes

## Transfer Functions and Transfer Characteristics

This document was prepared as review material for students in EE 230

By: Randy Geiger

Last Updates: Jan 16, 2010

Electronic circuits and electronic systems are designed to perform a wide variety of tasks. The performance requirements from task to task are often significantly different. Although the performance requirements vary widely, there are considerable benefits from both design and assessment viewpoints of having standard methods for characterizing the performance of these systems. The concepts of transfer characteristics and transfer functions are used extensively to characterize these circuits and systems.